2020

MATHEMATICS

[GENERAL]

Paper : II

[SUPPLEMENTARY]

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

GROUP–A

(Classical, Abstract and Linear Algebra)

(Marks : 50)

1. Answer any **two** questions: $1\times2=2$

 a) Show that the equation $27x^4-48x^2-12x+13=0$ has two positive roots in $(0,1)$ and $(1,2)$.

 b) Prove, without expanding, that

$$\begin{vmatrix} a & d & 3a-4d \\ b & e & 3d-4e \\ c & f & 3c-4f \end{vmatrix}=0$$

 c) Find the smallest positive integers n, if $\left(\frac{1+i}{1-i}\right)^n=1$.

2. Answer any **five** questions: $2\times5=10$

 a) Find the eigen values of the matrix $\begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}$.

 b) Find the equation whose roots are equal in magnitude but opposite in sign to the roots of $x^4+3x^3-7x^2+2x+1=0$.

 c) Prove that a second order skew-symmetric determinant is a perfect square.

 d) Let Z be the set of all integers. Is the mapping $f:Z\to Z$ defined by $f(n)=n^2+2$ a one-one mapping?

 e) Show that $x=1$ is a root of the equation

$$\begin{vmatrix} x+2 & 3 & 3 \\ 3 & x+4 & 5 \\ 3 & 5 & x+4 \end{vmatrix}=0.$$

 f) If $a, b\in G$, then prove that $(a.b)^{-1}=b^{-1}.a^{-1}$, where $(G,\bullet)$ is a group.

 g) Prove that $\sin(\log i^i)=-1$.

3. Answer any **three** questions: 6×3=18

a) If R be a ring and $a, b, c \in R$, then show that $-(a+b) = -a-b$.

b) Solve by Cramer's rule

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$3x+2y+9z=14$$

c) Show that the set of cube roots of unity is a finite abelian group with respect to multiplication.

d) State and prove De Moivre's Theorem for positive integer n only.

e) If one root of the equation $x^4-3x^3-5x^2+9x-2=0$ is $2-\sqrt{3}$, find the other roots.

4. Answer any **two** questions: 10×2=20

a) i) Show that $(2, 1, 4)$, $(1, -1, 2)$, $(3, 1, -2)$ form a basis of $V_3(F)$.

ii) Solve by Cardan's method

$$x^3-12x+65=0.$$ 5+5

b) i) If $x+\frac{1}{x}=\cos\frac{\pi}{7}$, show that $x^7+\frac{1}{x^7}=-2$.

ii) Solve $\begin{vmatrix} x+p & q & r \\ q & x+r & p \\ r & p & x+q \end{vmatrix}=0$. 5+5

c) i) Solve the system of equations $2x+3y+z=11$, $x+y+z=6$, $5x-y+10z=34$ by matrix method.

ii) Prove that every sub-group of a cyclic group is cyclic. 5+5

GROUP–B

(Analytical Geometry and Vector Algebra)

(Marks : 50)

5. Answer any **four** questions: 1×4=4

a) Determine the nature of the conic represented by the following equation:

$$x^2+6xy+9y^2-5x-15y+6=0$$

b) What will be the form of the equation $x^2-y^2=4$, if the axes are rotated through an angle $\left(-\frac{\pi}{2}\right)$?

c) If $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=2$, find $\vec{a}\times\vec{b}$.

d) Determine the nature and length of the latus rectum of the conic whose polar equation is $\frac{2}{r}=3-3\cos\theta$.

e) Find the value of 'm' for which the straight line $\frac{x-11}{3}=\frac{y-2}{m}=\frac{z+3}{-2}$ is parallel to the plane $x-3y+6z+7=0$.

f) Write down the equation of the bisector of the angles between the pair of straight lines $ax^2+2hxy+by^2=0$.

6. Answer any **six** questions: 2×6=12

a) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

b) Find the values of x and y for which the vectors $\vec{\alpha}=-3\vec{i}+4\vec{j}+x\vec{k}$ and $\vec{\beta}=y\vec{i}+8\vec{j}+6\vec{k}$ are collinear.

c) Prove that the straight lines $\frac{x-1}{3}=\frac{y+2}{1}=\frac{z+3}{2}$ and $\frac{x-5}{7}=\frac{y+8}{-5}=\frac{z-6}{11}$ are coplanar.

d) Find the equation of the plane passing through the point $(3,-4,1)$ and the line of intersection of the planes $4x-2y+3z-3=0$ and $2x+y+3z-1=0$.

e) Find the projection of the line segment joining the points $(3, 3, 5)$ and $(5, 4, 3)$ on the straight line joining the points $(2,-1,4)$ and $(0,1,5)$.

f) If the equation $6x^2+kxy-3y^2+4x+5y-2=0$ represents a pair of intersecting straight lines, find the value of k.

g) Find the radial axis of the circles

$$x^2+y^2+4x+8y+19=0 \text{ and}$$

$$x^2+y^2+8x+4y+19=0.$$

h) Find the co-ordinates of the centre of the circle $r=3\sin\theta+4\cos\theta$.

7. Answer any **four** questions: 6×4=24

a) Find the vector equation of the line joining the points $2\vec{i}-3\vec{j}-\vec{k}$ and $8\vec{i}-\vec{j}+2\vec{k}$.

b) Find the equation of the sphere for which the circle

$x^2+y^2+z^2+2x-4y+5=0$, $x-2y+3z+1=0$ is a great circle.

c) Find the image of the point $P(2,3,5)$ in the plane $5x+8y-z+16=0$.

d) Find the equation of the circle which passes through the origin and belongs to the co-axial system of whose limiting points are $(1, 2)$ and $(4, 3)$.

e) Reduce the equation $x^2+4xy+4y^2+4x+y-15=0$ to the canonical form and find the nature of the curve.

f) Show that the equation to the pair of straight lines through the origin perpendicular to the pair of straight lines $ax^2+2hxy+by^2=0$ is $bx^2-2hxy+ay^2=0$.

8. Answer any **one** question: $10\times1=10$

a) i) A variable plane which is at a constant distance 3p from the origin O cuts the axes in A, B, C. Show that the locus of the centroid of the triangle ABC is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}.$$

ii) A force $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of $\vec{F}$ about the point $(2, -1, 3)$. Also find the magnitude of it.

b) i) Find the equation to the right circular cylinder of radius 2 whose axis is the straight line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$.

ii) Prove by vector method that the diagonals of a parallelogram bisect each other.